## Fall 2013 Interstellar Problems

31. Two non-zero real numbers, a and b, satisfy ab = a - b. Find a possible value of  $\frac{a}{b} + \frac{b}{a} - ab$ .

(A) 
$$-2$$
 (B)  $-\frac{1}{2}$  (C)  $\frac{1}{3}$  (D)  $\frac{1}{2}$  (E) 2

- 32. Let A, M, and C be nonnegative integers such that A + M + C = 12. What is the maximum value of  $A \cdot M \cdot C + A \cdot M + M \cdot C + C \cdot A$ ?
- 33. One morning each member of Angela's family drank an 8-ounce mixture of coffee with milk. The amounts of coffee and milk varied from cup to cup, but were never zero. Angela drank a quarter of the total amount of milk and a sixth of the total amount of coffee. How many people are in the family?
- 34. When the mean, median, and mode of the list

are arranged in increasing order, they form a non-constant arithmetic progression. What is the sum of all possible real value of x?

35. Let f be a function for which  $f(x/3) = x^2 + x + 1$ . Find the sum of all values of z for which f(3z) = 7.

(A) -1/3 (B) -1/9 (C) 0 (D) 5/9 (E) 5/3

- 36. A checkerboard of 13 rows and 17 columns has a number written in each square, beginning in the upper left corner, so that the first row is numbered 1, 2, ..., 17, the second row 18, 19, ..., 34, and so on down the board. If the board is renumbered so that the left column, top to bottom, is 1, 2, ..., 13, the second column 14, 15, ..., 26 and so on across the board, some squares have the same numbers in both numbering systems. Find the sum of the numbers in these squares (under either system).
- 37. A circle centered at O has radius 1 and contains the point A. Segment AB is tangent to the circle at A and  $\angle AOB = \theta$ . If point C lies on  $\overline{OA}$  and  $\overline{BC}$  bisects  $\angle ABO$ , then OC =



(A) 
$$\sec^2 \theta - \tan \theta$$
 (B)  $\frac{1}{2}$  (C)  $\frac{\cos^2 \theta}{1 + \sin \theta}$  (D)  $\frac{1}{1 + \sin \theta}$  (E)  $\frac{\sin \theta}{\cos^2 \theta}$ 

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- 38. In year N, the 300<sup>th</sup> day of the year is a Tuesday. In year N + 1, the 200<sup>th</sup> day is also a Tuesday. On what day of the week did the 100<sup>th</sup> day of year N 1 occur?
- 39. In triangle ABC, AB = 13, BC = 14, and AC = 15. Let D denote the midpoint of  $\overline{BC}$  and let E denote the intersection of  $\overline{BC}$  with the bisector of angle BAC. What is the area of triangle ADE?
- 40. If x, y, and z are positive numbers satisfying

x + 1/y = 4, y + 1/z = 1, and z + 1/x = 7/3,

then what is xyz?